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PROBLEMS OF FLIGHT IN TURBULENCE

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16. Abstract

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PROBLEMS OF FLIGHT IN TURBULENCE

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SUMMARY

The paper deals with various problems raised by flight in turbulence. The first part is devoted to the description of mathematical models liable to provide the aircraft designer with a practical representation of the environment encountered by the aircraft; classical models are briefly recalled and new approaches, both British and French, are presented. The second section deals with the calculation of the response of a flexible aircraft to such an environment, and shows that, as a rule, the turbulence field should be considered as isotropic in this case. In the last part it is shown how active control systems open the way to a next generation of aircraft that will be less sensitive to gusts, and systems at present under test are described.

Introduction

After several decades of fundamental research, experimental and theoretical analysis, flight tests, and systematic examination of serious accidents, the problem of flight and turbulence continues to be the center of preoccupation of those responsible for aircraft projects, of directors of airlines, and of major countries.

This sustained interest is explained by two fundamental reasons:

(a) On the one hand, the great sensitivity which modern airplanes present to turbulence as a result of their characteristics and of their missions. In particular:

- For civil aircraft, the trend towards large weight and airspeed favors their response to the large wavelengths which are the most energetic.

- For military aircraft, ground attack missions or those of penetration at low altitude take place in a particularly severe environment of the atmospheric boundary layer.

(b) In the second place, the proliferation of problems where turbulence has an effect. For example:

- Definition of limit load factors and probability of encounter of destructive gusts;
- Estimation of the fatigue life, and frequency of inspection of the wing structure;
- Comfort of the passengers and crew of civil aircraft;
- The effectiveness in combat and maneuverability of military aircraft engaged in low altitude missions;
- The production of effective flight simulators, of automatic pilots, and of all-weather landing systems;
- Definition of the force output, bandwidth and authority limitations of servomechanisms;
- Integration into new aircraft of gust-alleviation systems.

The object of such a conference as this is evidently not to treat even briefly all the problems which have been stated, but rather to point out several scientific regimes indispensable to their development. The three fundamental themes considered are related to the following items:

- To the modeling of atmospheric turbulence;
- To the calculation or measurement of the transfer function of an airplane to turbulence;
- To the definition of optimal control methods for flight in turbulence.

Voluntarily excluded from this list are the meterological predictions of turbulence and the development of new damage criteria. These two themes correspond to disciplines which the author does not feel qualified to treat herein.

In the domain of modeling of atmospheric turbulence, after spectral methods have been rapidly recounted, we will indicate what are the tentative facts in comparing the point of view of discrete or continuous turbulence. In the following we suggest an approach capable of more accurately estimating the probability of encountering a gust capable of breaking the airplane; we will show how this approach has been utilized to apply statistics, unfortunately very scanty, of turbulence at high altitude to the case of supersonic flight of the Concorde.

The second part of the presentation will be devoted to the study of transfer functions of an airplane in turbulence. We will first recount the classical approaches in which the airplane is supposed to encounter cylindrical waves with generatrices perpendicular to the direction of flight. We will show the limitations of such an approach thanks to the notion of transverse correlation, and the methods of calculating the transfer function in isotropic turbulence will be established. Passing finally to the experimental point of view, we will treat the measurement of transfer functions in turbulence and some results obtained in flight on the Concorde.

The last chapter of the paper will consider the problem of automatic control of flight in turbulence. Two aspects will be described: closed-loop and open-loop systems, making clear in each case the limitations. The open-loop method in which the control motions are deduced from measurements of the turbulence encountered by the airplane are developed; and the theoretical predictions will be compared with results of flight tests on the Mirage III.

1 - Formulation of Atmospheric Turbulence

1.1 General techniques

Techniques for measuring atmospheric turbulence and methods for calculating the response of an airplane can be defined only if one first assumes a model of the atmosphere which will be compared with experiments and corrected little by little. In spite of the remarkable works conducted by meteorologists and aerothermodynamicians, obtaining a theoretical model seems actually still far away, if not illusory. In these conditions, one is led to a phenomenological representation making the analysis as simple as possible, on which one will impose the fundamental properties predicted by the theory. The principal information which theory furnishes relates to the form of the power spectral density of turbulence, a form which is represented schematically on figure 1.

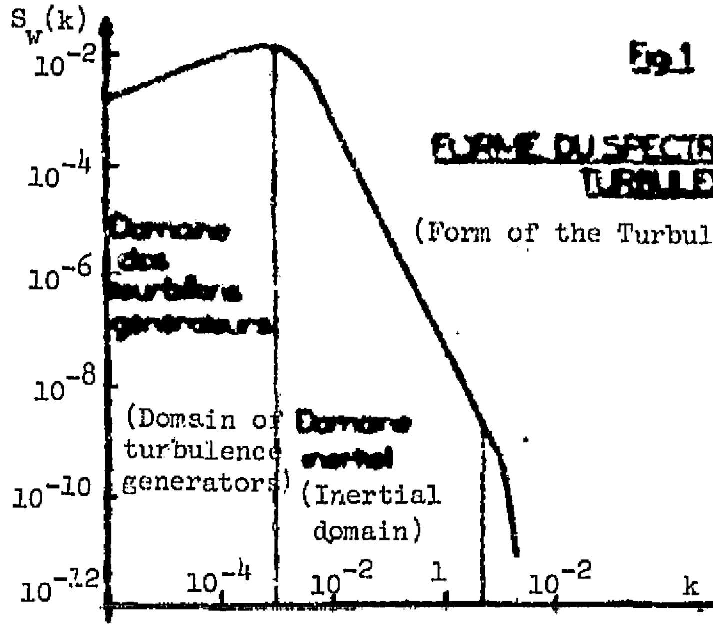


Fig 1

FORME DU SPECTRE DE LA TURBULENCE

(Form of the Turbulence Spectrum)

This spectral density presents three principal parts which correspond to different ranges of the wave number $k = \frac{1}{\Lambda}$. The first zone, which corresponds to very small values of k, that is to say, to very long wavelengths, is associated with the creation of great eddies which generate turbulence and probably cannot be made the object of any mathematical formulation. The second zone, covering a range of wavelengths from several thousands of meters to several centimeters, is called the inertial zone, for which the theories of Heisenberg or Kolmogorov predict a power law with exponent minus 5/3. The last zone, associated with large values of k, is that of small eddies which die out due to viscosity; Heisenberg calculated for this domain a power law with exponent minus 7.

The domain of wavelength of interest to aerodynamics goes from several meters to several thousand meters and corresponds, therefore, to the inertial omain of the theory. Since longer or shorter wavelengths are not felt by the airplane the phenomenological model is therefore imposed to have a power law with exponent minus 5/3 in the inertial domain and it is possible to neglect its disagreement with reality outside of this domain.

The theory furthermore assures that the turbulence is isotropic except in the neighborhood of the ground, where the effects of the boundary layer are predominant. This condition should also be imposed on the model. Finally, one admits the hypothesis of Taylor (1), well verified by experiments, which considers the atmosphere as frozen during the time of passage of an airplane. This indispensable hypothesis permits one to associate frequencies with wavelengths encountered by an airplane which flies at the speed V by the relation:

$$(1) \qquad \Lambda = VT = \frac{V}{f}$$

Other pieces of information are also furnished by the theory, for example, the stability of the turbulence determined by the Richardson number, but this information has no interest in the discussion of the statistical model which ignores the meteorological conditions.

In the general case, we have stated the tentative assumptions to describe statically the atmospheric turbulence. Models which we will present consider successively the turbulence first as a continuous process, next as a collection of discrete gusts, and finally, as a collection of "packets" having a Poisson distribution.

1.2 Continuous model of turbulence

The continuous model of turbulence which is now in current use for certification is due to Clementson, Reference (2), and Press, References (3), (4), and (5). In this model one considers that the atmosphere is formed of stationary and Gaussian "packets" of turbulence, which all have spectral densities of the same form and differ only in their variance.

To describe statistically the turbulence, therefore necessitates two operations:

- The choice of spectral density of the local processes.
- The definition of probability that a process will have the variance σ^2.

We will treat successively these two aspects.

(a) Representation of a local process

To characterize a local process, it is considered to be Gaussian with a power spectral density represented by the model of Karman. The longitudinal component u, and the transverse components v and w of the turbulence (figure 2) have, respectively, the spectral densities as shown in formula 2.

$$
(2)\begin{cases}
\tilde{S}_u(k) = \dfrac{2\sigma_u^2 L}{\left[1+(1.339 \times 2\pi k L)^2\right]^{5/6}} \\[2ex]
\tilde{S}_w(k) = \tilde{S}_v(k) = \sigma_w^2 L \, \dfrac{1+8/3\,(1,339 \times 2\pi k L)^2}{\left[1+(1,339 \times 2\pi k L)^2\right]^{11/6}}
\end{cases}
$$

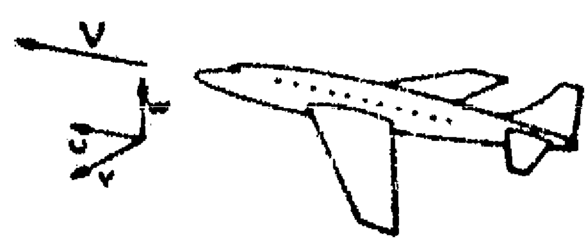


Fig.2 COMPOSANTES DE LA TURBULENCE ATMOSPHERIQUE
(Components of the Atmospheric Turbulence)

It is noted that spectral densities of v and w are indeed in the inertial domain, a region with a power law of exponent minus 5/3. Taking into account Taylor's hypothesis given by formula (1), each turbulence region appears to the airplane which is flying at the speed v as a temporal process of spectral densities:

$$
(3)\begin{cases}
S_u(\omega) = \dfrac{\sigma_u^2 L}{\pi V} \, \dfrac{1}{\left[1+\left(1,339\,\dfrac{\omega L}{V}\right)^2\right]^{5/6}} \\[3ex]
S_w(\omega) = S_v(\omega) = \dfrac{\sigma_w^2 L}{2\pi V} \, \dfrac{1+8/3\left(1,339\,\dfrac{\omega L}{V}\right)^2}{\left[1+\left(1,339\,\dfrac{\omega L}{V}\right)^2\right]^{11/6}}
\end{cases}
$$

In these expressions, σ_u^2 and σ_w^2 represent, respectively, the variances of the longitudinal and transverse components and L a reference length called the macroscopic scale of turbulence. Values proposed by different authors, References (6) and (7), of the scale L vary, which fact is of little importance except at very low altitudes, because the scale does not influence the form of the spectrum except at very large wavelengths, outside the inertial domain to which the airplane is sensitive. For the low altitudes, less than 500 meters, the scale is generally considered to equal 1/2 the altitude Z of flight:

$$L = \frac{Z}{2}$$

Since the local process is assumed Gaussian, the average number of zero crossings per second of the transverse component with a slope from positive to negative is given by the first formula of Rice, Reference (8):

$$(4) \qquad N_{ow} = \frac{1}{2\pi\sigma_w}\left[\int_0^{+\infty} \omega^2 S_w(\omega)\, d\omega\right]^{1/2}$$

The average number of exceedances of value x is given by the second formula of Rice:

$$(5) \qquad N_w(\sigma_w, x) = N_{ow} \exp\left(-\frac{x^2}{2\sigma_w^2}\right)$$

The choice of a Gaussian law for the local processes is the weak point of the representation, as a result of the fact that the probability densities measured in flight are much closer to an exponential law than to a Gaussian law. This remark does not invalidate the general principles, for it has been shown, (Reference 0), that the formulas of Rice may be extended to nonGaussian stationary processes having a reduced probability density f(x) with the form:

$$N_w(x) = N_{ow} \frac{f\left(\frac{x}{\sigma_w}\right)}{f(0)}$$

and N_{ow} is again given by formula (4).

In accordance with custom, in spite of everything that has been said, the following discussions will follow a locally Gaussian representation of turbulence.

(b) Global representation

The global representation consists in giving the probability density $\rho(\sigma_w)$ of the different encounters of turbulence, that is to say a proportion of time of flight passed in "packets" of turbulence of variance σ_w^2. For the entire atmosphere the average number of crossings per second of the value x by the vertical component w of turbulence is then given by:

$$N_w(x) = \int_0^{+\infty} p(\sigma)\, N_w(\sigma, x)\, d\sigma$$

That is to say, since N_{ow} is independent of the variance, by:

$$(6) \quad N_w(x) = N_{ow} \int_0^{+\infty} p(\sigma) \exp\left(-\frac{x^2}{2\sigma^2}\right) d\sigma$$

The density $p(\sigma)$ of the encounters is chosen to represent in the best possible way the body of results obtained from V.G.H. recorders (Velocity, Gravity, and Height) obtained in the course of thousands of hours of flight. As a result of actions taken in the series of projects "all-cat" and of cooperative programs of AGARD, References (19), (11), and (12), agreement seems to be reached on a representative bi-Gaussian probability $p(\sigma)$ of the form:

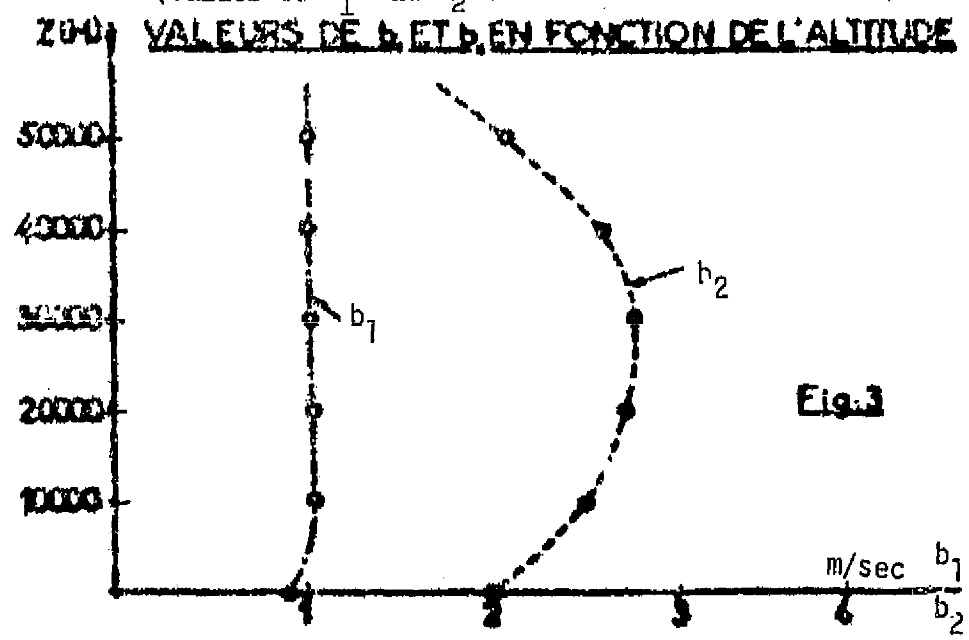
$$(7) \quad P(\sigma) = \left(\frac{2}{\pi}\right)^{1/2} \frac{P_1}{b_1} \exp\left(-\frac{\sigma^2}{2 b_1^2}\right) + \left(\frac{2}{\pi}\right)^{1/2} \frac{P_2}{b_2} \exp\left(-\frac{\sigma^2}{2 b_2^2}\right)$$

Integration of formula (6) can now be made explicitly, and leads to the result:

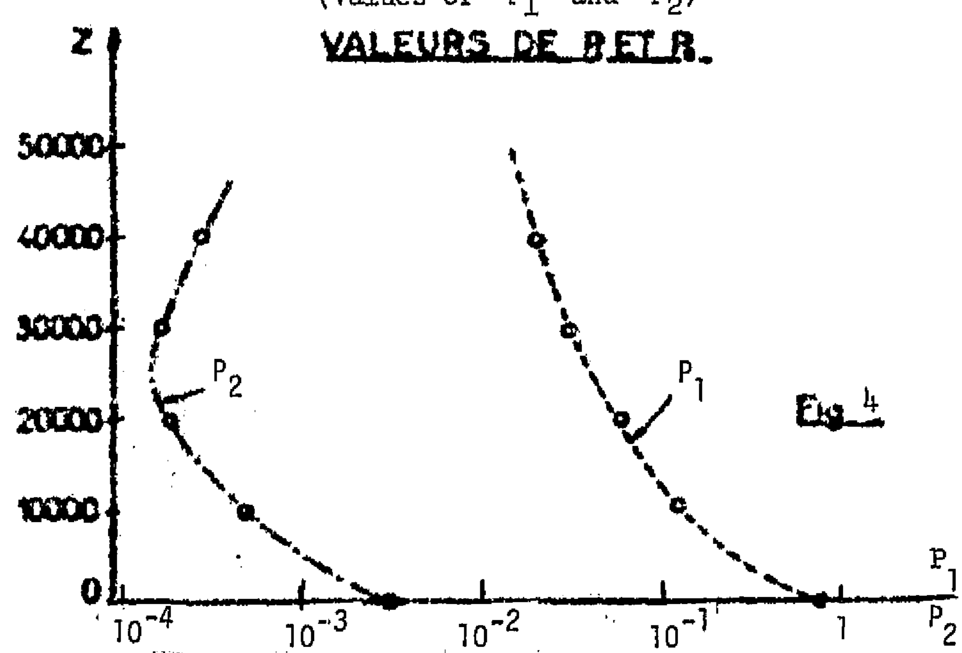
$$(8)\quad N_w(x) = N_{ow}\, P_1 \exp\left(-\frac{x}{b_1}\right) + N_{ow}\, P_2 \exp\left(-\frac{x}{b_2}\right)$$

Values of coefficients P_i and b_i actually proposed by the FAA are given, as the function of altitude, in figures (3) and (4):

(Values of b_1 and b_2 as a Function of Altitude)



(Values of P_1 and P_2)



The model is now complete and represents with sufficient fidelity the results obtained in flight. Let the author, however, be permitted to emphasize his contribution. The introduction of stationary and Gaussian "packets" of turbulence is perhaps satisfying to the spirit, since practically every passenger has remarked that the turbulence is presented in "packets." That idea is not, however, totally factual. In practice no one has ever measured the distribution $p(\sigma)$ of the different levels of turbulence. That has only been calculated, after the fact, in a way to well represent the result that the statistics of the ensemble have the form of the combination of two exponentials. In these conditions, it is sufficient to start off "a priori" with formula (8) as representative of the exceedences of the turbulence and choose the values of P_i and b_i in the manner to best represent the experimental results.

1.3 - Model of discrete turbulence

Well before the models which we have described in paragraph 1.2, the first studies of response of airplanes in turbulence were based on the concepts of isolated gusts produced by chance to which one assigned a priori the form of the probability density. One of the steps of certification of the airplane is still today the calculation of response to an isolated gust of which the form may be trapezoidal, one part of a cosine wave, etc., following the custom of the responsible authority. The interest in the concept of isolated gusts results from the fact that if, in the majority of cases, turbulence appears as a continuous process, exceptional circumstances exist wherein very strong turbulence presents itself as an isolated phenomenon. Calculation of the response of the airplane to a discrete gust permits definition of the history of loads and of stresses in these particular conditions, and drawing of conclusions on the capability of the structure.

The concept of discrete gusts suffers, however, from a fundamental difficulty, related to the arbitrary form of the chosen gust, which does not permit taking into account the distribution of energy among the different wavelengths; a distribution, as we have seen, imposed by theoretical considerations and well verified by experiments. The work recently carried out by Jones, References (13), (14), and (15), of the RAE looks toward eliminating this difficulty by introducing a distribution of discrete gusts of which the form and amplitude are related in such a way that the ensemble of events represents a process of which the spectral density agrees, in the inertial domain, with a power law of exponent minus 5/3.

To do this, Jones considers gusts of the form represented in figure (5):

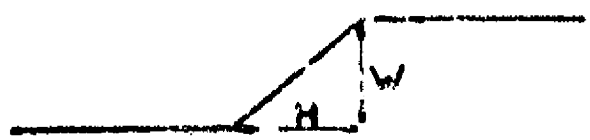


Fig. 5

MODELE DE RAFALE DISCRETE
(Model of Discrete Gust)

That is to say, in fact, half gusts with a perturbation velocity W developed over a length H. He then postulates that the average number of gusts per unit of length which start in the interval between H and H + dH and which have a maximum intensity of W, is given by $N_{W,H}dH$ where:

$$(9) \quad N_{w,H} = \frac{\alpha}{H^2} \exp\left(-\frac{W}{k H^{1/3}}\right)$$

He finally shows that a spectral density having a power law with exponent minus 5/3 in the inertial domain corresponds to a process formed by an assembly of such gusts. Introducing the notion of a gust tuned to each mode of an airplane, that is to say that each mode only responds to a relatively narrow band of values of H, he finally elaborates a coherent scheme susceptible to representing well the response statistics of an airplane in turbulence conforming with experimental results for a combination of exponentials.

The model proposed by Jones, although attractive, also presents some inconsistencies.

- First, the arbitrary choice of the form of the gust: it is probable that one would have been able, by starting out with other forms, and with conditional probabilities relating the intensity to a form parameter, to arrive at a correct spectral density of the turbulence.

- Next, and most importantly, one should hope in view of formula (9) that the exceedences of the turbulence, conforming to experiments, would have an exponential representation. This is not true, since the average number of exceedences of a value W, for the ensemble of gusts, is given by:

$$N_W = \int_0^{+\infty} N_{W,H}\, dH$$

corresponding to a law of W^{-3} (the law is no longer exponential when the integration is limited to values of H which corresponds to the inertial domain).

In spite of these few negative remarks, the author wishes to repeat his interest in the approach of Jones which tends for the first time to unify the discrete approach and the spectral approach and which will surely be the basis of new developments.

1.4 - Research on a new model

The continuous models and discrete models of turbulence which have just been described in the preceding paragraphs are of practical use and are therefore the object of routine calculations as an essential part of certification; they also serve as the basis of the establishment of load spectra utilized during fatigue tests. The estimation of extreme loads is also made thanks to these models, but evidently suffers from the small confidence which is generally accorded to limiting values of an experimental distribution.

To alleviate these difficulties, certain authors, among them, Buxbaum, Reference (16), are investigating an extrapolation model based on the log-normal distribution of extreme values. We suggest for our part a new approach, based on a Poisson model.

The establishment of the model rests on the following remarks: that the turbulence, whenever it is encountered, generally has the appearance of a sufficiently strongly correlated continuous phenomenon, the encounters with the turbulence are sufficiently rare, and the existence of violent turbulence is exceptional. In these conditions the appearance of a packet of turbulence is probably independent of the preceding appearance of another packet and one will be led to consider that the process is a Poisson process. To be more precise, it will be assumed that the event "existence of a packet of turbulence with intensity greater than x" obeys a law of Poisson.

In these conditions, calling $\mu(x)$ the average number of events per unit of length, the probability that k events will be encountered over a distance d may be expressed by:

$$(10) \quad P_d(K;x) = \frac{(d\mu(x))^k}{K!} \exp(-d\mu(x))$$

The average value $\mu(x)$ of the process will evidently depend on the intensity x; this will be determi. . on the assumption that, for strong turbulence, the extreme amplitudes will appear only once in each packet. $\mu(x)$ will in consequence be equal to $N_w(x)$ for the large values of x, as follows:

$$(11) \quad \mu(x) = P_2 N_{0w} \exp\left(-\frac{x}{b_2}\right) \qquad \text{for} \qquad x \gg b_1$$

Formula (10) may be expanded to:

$$(12)$$

$$P_d(k;x) = \frac{d^k}{K!} (P_2 N_{0w})^k \exp\left(-\frac{kx}{b_2}\right) \exp\left(-d P_2 N_{0w} \cdot \exp\left(-\frac{x}{b_2}\right)\right)$$

It is therefore possible to estimate, thanks to this model, the possibility of encountering a packet of turbulence, that is to say, a region of turbulence with a value greater than x, over a distance d greater than that which has been used for the establishment of the statistics. The probability $P'(d\ ;\ x)$ is given by the expression:

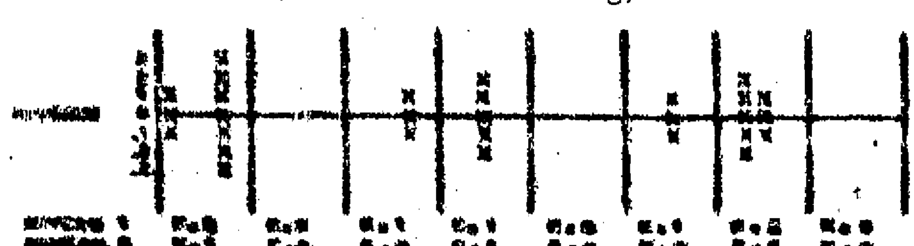
$$P'(d;x) = 1 - P_d(0;x) = 1 - \exp\left(-d P_z N_{ow} \exp\left(-\frac{x}{b_z}\right)\right) \qquad (13)$$

Passing to the limit for very small probabilities, it is possible to verify after the fact the formula currently employed by aircraft constructors:

$$(14) \quad P'(d;x) \approx d P_z N_{ow} \exp\left(-\frac{x}{b_z}\right)$$

The validation of the model has been tried in the case of the analysis of the turbulence encountered by the Concorde, in supersonic flight at high altitude corresponding to a total of about 54,000 km. The number of exceedences has first been counted, positive and negative, of six values of turbulence separated by 1.13 meters/second, for consecutive samples of 300 km. The number of regions of 1,800 km. for which k samples have presented a turbulence greater than the value x; have then been found. One has thus determined starting out with the measurements, the frequencies P_d (k ; x). Figure 6 represents schematically the organization of the workup.

(Method of Subdividing)



METHODE DE DEPOUILLEMENT

Sur cet exemple. $P(0,1) = \frac{3}{8}$; $P(1,1) = \frac{3}{8}$; $P(2,1) = \frac{2}{8}$
(In this example)
$P(0,2) = \frac{2}{8}$ $P(1,2) = \frac{3}{8}$; $P(3,2) = 0$

Fig. 6

Given a value x, the value of $\mu(x)$ from equation (10) has been determined. This has been done for each value of k. Table 1 summarizes the results obtained.

(Experimental Values of $\mu(x)$)

VALEURS EXPÉRIMENTALES DE $\mu(x)$

Tableau 1

(Table 1)

$W > 1{,}13$ m/sec			$W > 2{,}26$ m/sec			$W > 3{,}39$ m/sec		
k	$P(k,x)$	$\mu(x)$	k	$P(k,x)$	$\mu(x)$	k	$P(k,x)$	$\mu(x)$
0	0,3666	1,000	0	0,7167	0,333	0	0,9167	0,087
1	0,3000	1,770	1	0,2500	0,334	1	0,0833	0,093
2	0,2333	1,350	2	0,0333	0,300	2		
3	0,0500	0,930	3			3		
4	0,0333	1,300	4			4		

Following a large dispersion for the level 1, the results for the levels 2 and 3 show that $\mu(x)$ is practically constant for each level, which confirms the Poisson model proposed. The last step consists in studying the appearance of the function $\mu(x)$; the result shown in figure 7 on a semilogarithmic scale shows that a $\mu(x)$ is practically exponential.

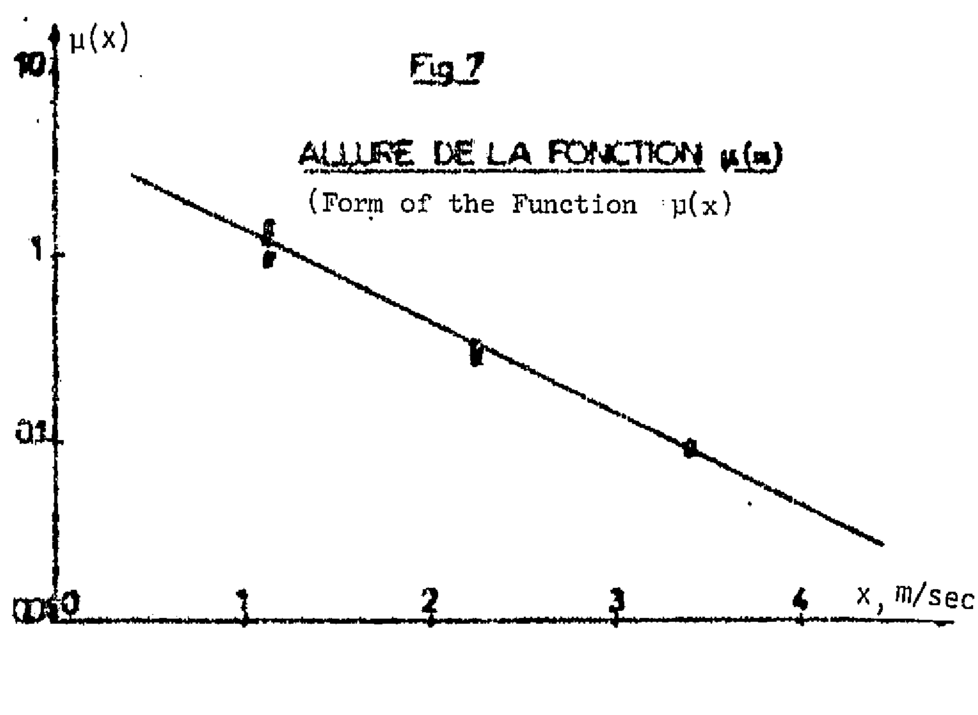


Fig 7

ALLURE DE LA FONCTION $\mu(x)$

(Form of the Function $\mu(x)$)

Finally, for the flights of the Concorde at high altitude, the Poisson model, with respect to kilometers travelled, is given by the expression:

$$(15) \quad P_{kw}(k,x) = \frac{(2{,}6 \cdot 10^{-3})^{k}}{k!} \exp(-1{,}17 k x) \exp\left(-2{,}6\ 10^{-3} \cdot \exp(-1{,}17 x)\right)$$

It is well to point out at the end of this paragraph that the model presented is unfinished; it will be necessary to complete it to determine the statistical distribution in the interior of each section. Problems of spectral density, on the other hand, do not arise. We will assume quite naturally that the spectral density of Karman is representative.

2 - Transfer Function of an Airplane in Turbulence

2.1 - Introduction

Once a model of the atmosphere is chosen, the next step of the engineer's work consists of predicting the behavior of the airplane and, more precisely, its flying qualities in turbulence, loads induced by the gusts, the stresses imposed by these loads, and, by the introduction of statistics, risks of encountering catastrophic loads and the length of the fatigue life. It is required in most cases, as a routine job carried out at the end of certification, which eventually depends on the knowledge of the transfer function of the airplane in turbulence.

In the contents of this paper, we will limit ourselves to two aspects of the problems placed by the determination of transfer functions:

- In the theoretical area, on the critical analysis of calculation methods based on the notion of gusts which are uniform across the span - methods which are generally utilized by aircraft constructors.
- In the practical area, to the description of a method of measuring in flight the transfer functions of an airplane in turbulence; these measurements in practice prove necessary to correct calculations which are too often not very representative of the actual behavior of the airplane. Finally, the results obtained in flight on the Concorde will be presented.

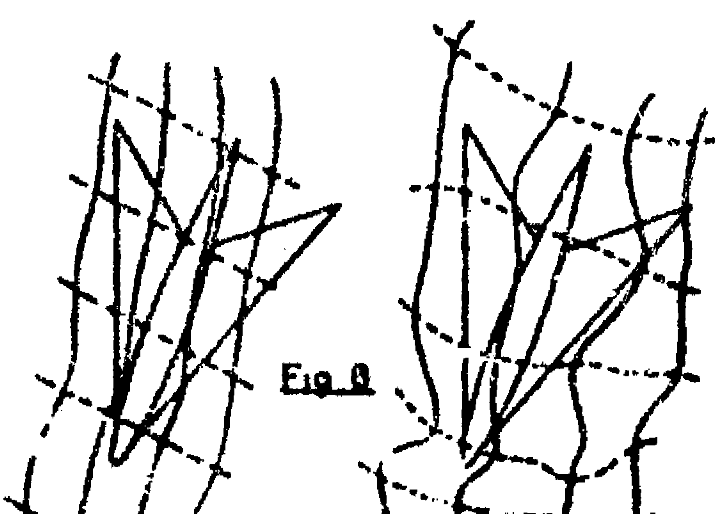
2.2 - Appreciation of the influence of isotropy of the turbulence

The general study of transfer functions in a three-dimensional field of turbulence has been remarkably treated in a recent publication of Houbolt, Reference 17. The author shows as a result of considerations of order of magnitude that the study of the transfer functions may be conducted separately for longitudinal and lateral responses. For the longitudinal response, the vertical component of turbulence is the only one important, while the transverse gusts are the only ones responsible for the lateral responses. With regard to longitudinal gusts directed along the axis of flight Houbolt shows that these have only small importance.

We will attempt here to appreciate the influence of the isotropy of the turbulence on the calculation of longitudinal response of an airplane, Reference 18. In connection with such a calculation, two hypotheses may be made regarding the structure of the field of turbulence encountered by the airplane (Figure 8):

- Whether indeed one considers that the airplane encounters turbulence constant across the span, that is to say, formed of cylindrical waves with generatrices normal to trajectories; that is the hypothesis made very generally by the aircraft constructors with the object of simplification.
- Whether one presumes, which corresponds better to the physical reality, that the turbulence has no more reason to be constant along the span than along the flight path. One is then led to a scheme of isotropic turbulence.

(Turbulence Uniform Across the Span)



(Isotropic Turbulence)

Fig. 8

Turbulence uniforme en envergure

Turbulence isotrope

To appreciate the influence of isotropy we will calculate the transverse correlations function associated with the common model of isotropic turbulence. If one calls $g(r)$ the transverse correlation function of isotropic field of turbulence, one may show without difficulty (Reference 18), that the gust correlation function between the vertical components measured at two points M and M' of a wing with a difference in time τ (Figure 9) is given:

$$R_w(M,M',\tau) = g\left(\sqrt{\left(V\tau-(x-x')\right)^2+(y-y')^2}\right) \tag{16}$$

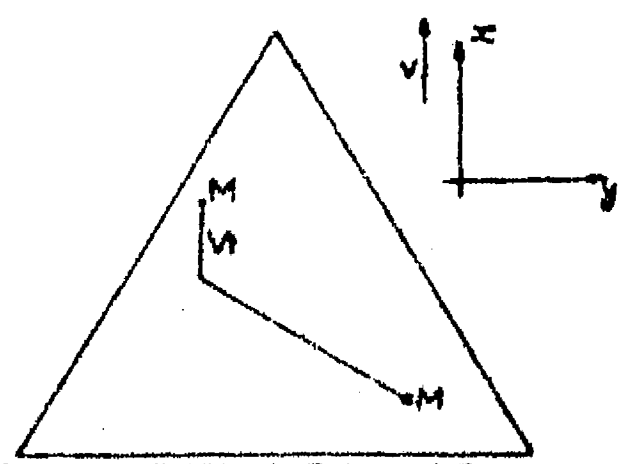


Fig 9 PROPAGATION DE L'INFORMATION SUR UNE AILE

(Propagation of Information on a Wing)

By Fourier transformation one may deduce the cross spectral density:

$$S_w(M,M',\omega) = S_w(\omega)\, C\left(\omega, \frac{|y-y'|}{V}\right) \exp\left(-i\,\omega\, \frac{x-x'}{V}\right) \tag{17}$$

Where $Sw(\omega)$ is the common spectral density presented in formula 3 and $C(\omega\ ;\ \xi)$ is the transverse correlation function which causes the loss of correlation across the span. This function is given by the expression:

$$(18)\quad C(\omega;\eta) = 1 - \frac{|\eta|}{S_w(\omega)} \int_0^{+\infty} S_w\left(\sqrt{\omega^2+v^2}\right) J_1(|\eta| v)\, dv$$

For the model of Karman and for the frequencies associated with the inertial domain, $C(\omega\ ;\ \eta)$ may be expressed in the form:

$$(19)\quad C(\omega;\eta) = \frac{2}{\Gamma\left(\frac{5}{6}\right)} \left(\frac{\omega\eta}{2}\right)^{5/6} K_{5/6}(\omega\eta)$$

where $K_v(x)$ represents a modified Bessel function of the second kind of order v.

The next step consists of calculating the transverse correlation length:

$$\ell = \int_0^{+\infty} C\left(\omega, \frac{y}{V}\right) dy$$

Then the integral is evaluated by substituting expression 19, and one obtains:

$$(20)\qquad \ell = 1{,}403\,\frac{V}{\omega}$$

It is noted then that each time the wavelength ℓ associated with a speed v and a vibration mode ω is very large compared to the span b, the hypothesis of velocity constant across the span will be acceptable and will lead to reasonable results. On the contrary, if the parameter

$$p = \frac{\ell}{b} = 1{,}403\,\frac{V}{b\omega}$$

is of the order of magnitude of one or less, the hypothesis of constant velocity across the span will not be admissible, and one must take into account the isotropy of the field of turbulence.

Table 2 indicates the values of the parameter p for the pitch mode and for the first bending mode of four recent airplanes in the cruise condition. It appears for the four examples that the hypothesis of constancy across the span is reasonable in the domain of frequencies associated with flight mechanics and unacceptable for the calculation of response in the domain associated with elastic modes. We will assume that this conclusion is general.

(Typical Values of the Parameter p)

VALEURS TYPIQUES DU PARAMETRE P

(Table 2)

TABLEAU 2

(Type of Airplane) TYPE D'AVION	(Pitch Mode) MODE DE TANGAGE	(First Wing Bending) PREMIERE FLEXION VOILURE
CARAVELLE	4	1
B707	5	0.8
CONCORDE	10	1.3
B747	7	1.1

The critical analysis which has just been developed leads very quickly to counseling the constructors of aircraft to carry out their calculation of transfer functions of flexible airplanes in turbulence by use of the hypothesis of isotropy. It is useful to recall on this occasion that it has been shown that calculations made under these conditions hardly present any additional complications.

To illustrate the influence of isotropy, we present in Figure 10 a comparison between the transfer functions to turbulence calculated for a Mach Number of 0.8 which follows from the two hypotheses.

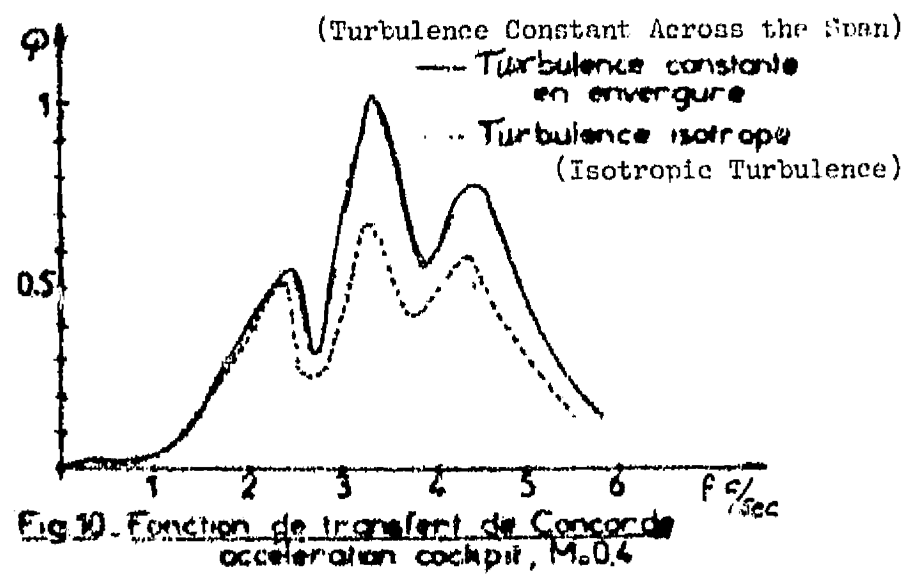


Fig. 10.- Transfer Function of the Concorde,
Cockpit Acceleration, M = .4

2.3 Measurement of transfer functions to turbulence

No matter how carefully the calculations of transfer functions of turbulence are made, the comparison between predictions and measurements is often deceiving. The reasons for disagreement are quite numerous. It is not sufficient, as in oscillation problems, to make a study of stability in which one is interested only in the roots of the characteristic equations, rather one must make a calculation of a forced response, much more sensitive to all the errors which can be committed in the appreciation of the modes of the structure and the aerodynamic forces. In particular, precise determination of damping, both structural and aerodynamic, is indispensible.

Flight tests then appear necessary to validate or to correct the calculation of transfer functions. Like many organizations the ONERA has developed and utilized a method of measurement of transfer functions (Reference 20). The method is as follows: the turbulence encountered by the airplane is measured in real time from the data obtained from a vane (incidence α), from a rate gyro (pitching velocity $\dot{\theta}$) and from an accelerometer (acceleration $\ddot{z}$), by analog solution following from the kinematic equation:

$$\alpha = \theta - \frac{\dot{z}}{V} - \frac{w}{V}$$

One assures oneself that the measurement is correct by verifying that it gives a null result during piloted maneuvers in calm air.

The turbulence thus measured is recorded on magnetic tape simultaneously with the response parameters (accelerations, stresses, . .) for which one wishes to know the transfer functions. Since the disturbance w and the response (for example, acceleration $\ddot{z}$) are known, one determines the transfer function $T_{\ddot{z}_w}(\omega)$ by the classical identification techniques.

To do this, one proceeds, on the ground, to the calculation of direct and cross-spectral densities:

$S_w(\omega)$, spectral density of turbulence,

$S_{\ddot{z}}(\omega)$, spectral density of the response,

$S_{\ddot{z}w}(\omega)$, cross-spectral density between the acceleration and the turbulence.

One deduces from this, by the classic formula for linear transfer between random processes, the transfer function $T(i\omega)$ of the parameter considered to turbulence:

$$T(i\omega) = \frac{S_{\ddot{z}w}(\omega)}{S_w(\omega)} \tag{21}$$

Work is being pursued at ONERA, in research, by a least squares method, of an explicit mathematical model of the experimental transfer function in the form of a rational fraction (Reference 21), a model which one will be able to compare directly with theoretical predictions.

In order to illustrate the quality of results which may be obtained, we present in figure 11 the transfer functions measured in flight in the approach conditions for the acceleration of the pilots station. The figure presents (from top to bottom) the modulus of the transfer function (a), and its real imaginary parts (b) and (c).

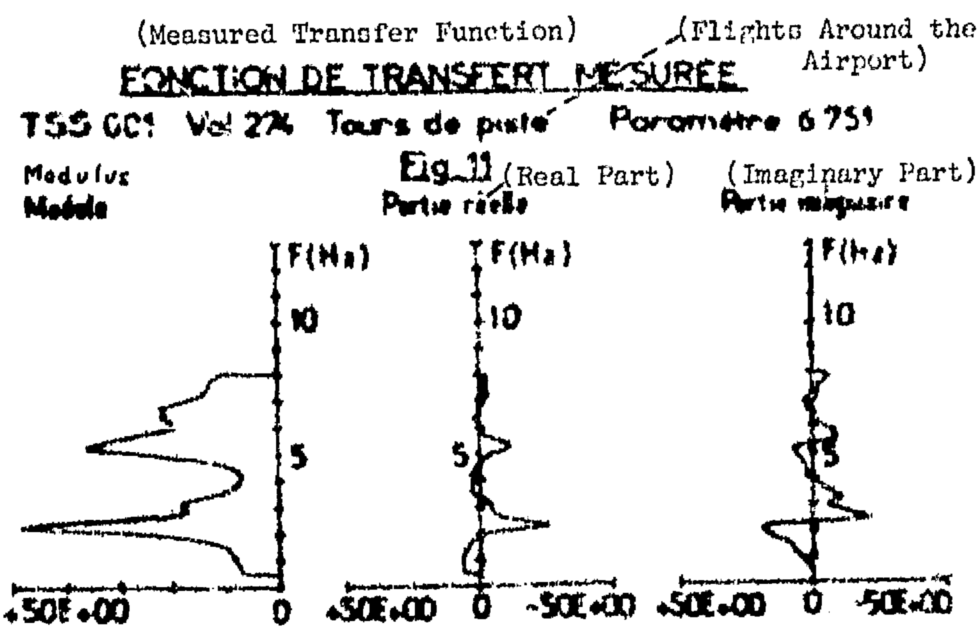


Fig. 11

Figure 12 presents the same transfer function represented in polar coordinates (real part on the abscissa, imaginary part on the ordinate), as well as its fitting by rational fractions. The author regrets not presenting any comparisons between calculated and measured transfer functions; this results from fact that the rare turbulence encounters in flight with the Concorde do not correspond with the calculated points. The calculations for comparison are in progress.

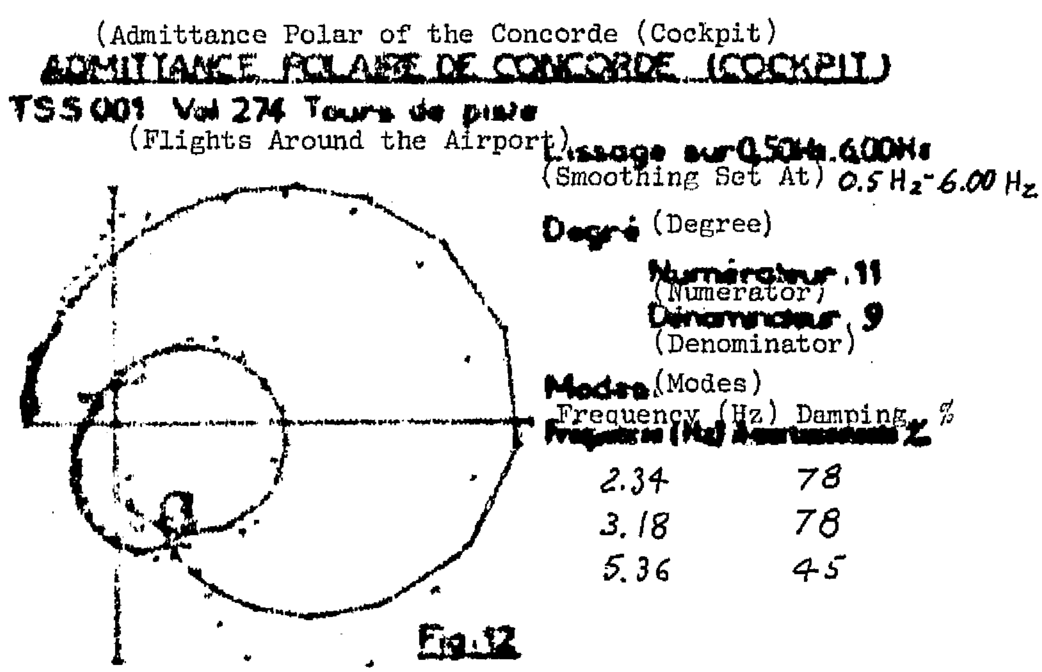


Fig. 12

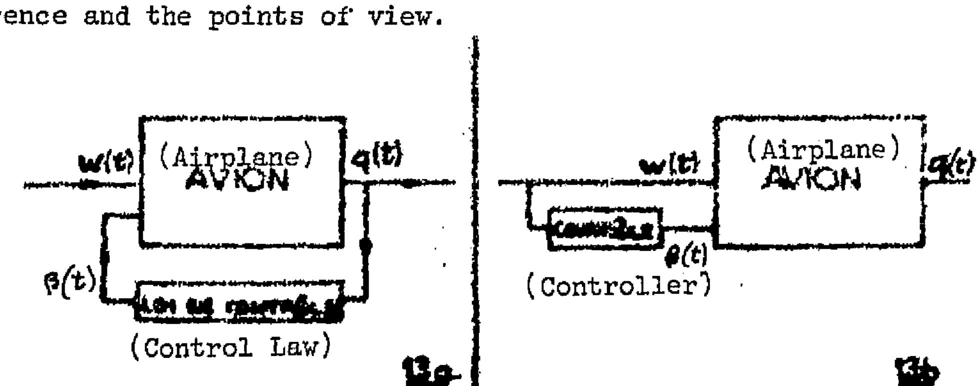
In conclusion, comparisons may be made between calculated and measured transfer functions on other airplanes, with the conclusion that the agreement is good in the frequency domain associated with mechanics of flight but degrades very rapidly at high frequencies. As the large stresses are often associated with higher order modes, it is possible to appreciate the importance for prediction of duration of fatigue life of the problems imposed by these disagreements. The simplified hypothesis of turbulence constant across the span is probably responsible for one part of the difficulties, but does not permit the complete explanation of the poor representativity of the calculations. A vast field of research therefore remains throughout this domain.

3 - Optimization of Flight in Turbulence

3.1 Posing of the problem

The last part of the presentation is devoted to a problem which has assumed more and more importance in recent years, the problem of optimum control of flight in turbulence. This work which is included in the general field of CCV studies (controlled configured vehicles) has numerous motivations. These include the necessity of increasing fatigue life, of improving the comfort of passengers and crew, of stabilizing the gun platform at low altitudes, etc. The interest in decreasing the response of the airplane in turbulence is therefore evident.

Two schools of thought which have different approaches to the problem have been developed in recent years. The first represented essentially by NASA and Boeing presumes closed-loop systems, while the second, of ONERA, looks to the improvement of control in an open-loop manner. Figure 13 illustrates the difference and the points of view.



(Principles of Control of Flight in Turbulence)
Fig.13 PRINCIPES DE CONTRÔLE DU VOL EN TURBULENCE

Part (a) of the figure sketches a closed-loop system: the airplane under the action of turbulence $w(t)$ produces responses $q(t)$ which are measured and reinjected through a control law to the controls of the airplane in a way to minimize the variation of the response.

In the open-loop system, part (b) of the figure, the turbulence encountered is measured in real time onboard the airplane by means of information from a vane, gyroscope, and an accelerometer. It is this signal $w(t)$ which by the intermediary of an appropriate filtering law operates the controls. In the first type of system, the entire behavior of the airplane is modified, in particular, its flight mechanics, its reactions to the commands of the pilot, etc. In the second, on the contrary, all the properties and qualities of flight are unchanged with the exception of the transfer functions in turbulence. After having described briefly the closed-loop method and the remarkable applications which it has had in the United States in the LAMS Program, we will develop the principles of the method studies at ONERA, and we will show on a simple example its interest and limitations.

3.2 - <u>The optimization in the closed-loop system</u>

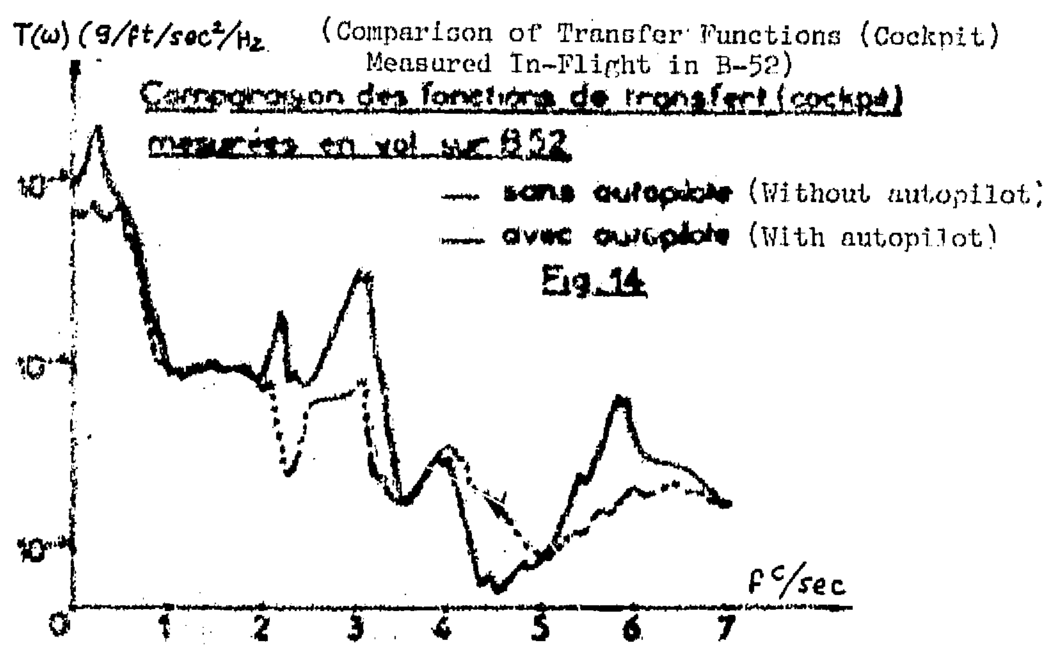
First, in the case of the LAMS Program and more recently in a more general case of CCV systems, NASA and Boeing Wichita have undertaken numerous studies of optimization of flight in turbulence, (11), (22), (23), and (24). In all cases, the principle rests on utilization of a regulating network which operates a complex system of controls in response to accelerometer measurements on the structure. Characteristics of the network are chosen in a way to favorably modify the transfer functions in turbulence of certain critical responses.

In the process of definition of the system for a B-52, 30 elastic modes have been introduced in the calculation of symmetric movements, 27 in the calculation of antisymmetric movements. The effects of limitations of deflection imposed on the controls have been taken into account and it has been verified by analog studies that the responses of the airplane remain sufficiently Gaussian for the formulas of Rice to be applicable. The model of the atmosphere chosen was the model of Press, each local process being described by a Dryden spectrum. Since the chosen system requires utilization of servomechanisms with large gain and large band pass, particularly delicate studies of stability had to be conducted.

The theoretical studies were completed in a wind tunnel on a model dynamically similar to the airplane, equipped with a complete system of controls and excited by a periodic disturbance induced by angle of attack generating oscillating vanes. Boeing has produced on this occasion remarkable electrohydraulic servomechanisms which can be located in the model.

The airplane equipped with horizontal and vertical canards was then made the object of a first series of flight tests in the open-loop condition with sinusoidal oscillation by the canards. The transfer functions of this exitation were measured and compared with theoretical predictions with the object of making corrections to the mathematical model of the airplane.

The last step of the work during 1973 consisted of 9 hours of flight in turbulence, at low altitude, in the course of which were measured the transfer functions to turbulence with and without the control system. Figure 14 taken from reference 24 presents the transfer functions. The improvement obtained due to the employment of the system, that is to say, the ratio of the variance of the basic airplane to the variance of the airplane with the autopilot, is of the order of three.



The excellent results obtained fit into a very general class of American policy oriented towards the utilization of active controls both for flight in turbulence and for the augmentation of critical speed of flutter or the reduction of maneuver loads. Already, the Supersonic Transport Project of Boeing has made a large utilization of these techniques and it is known that the bombing airplane B-1 will be equipped with a system for controlling flight in turbulence.

3.3 Open-loop optimization

Although it is perfectly logical when considering a general philosophy of CCV to assign the control of flight in turbulence to a counter-reacting system, such a system has important limitations particularly in the domain of flight mechanics. In effect, the action of a closed-loop system consists essentially in damping out the airplane modes, which is in all cases favorable for the flexible modes, but may lead to a deterioration of the handling qualities associated with the rigid modes; but it is just in the domain of frequencies associated with mechanics of flight that the turbulence presents its maximum energy. One is thus lead to conceive a mixed system where the counter-reacting method is adopted for the flexible modes but an open-loop method is chosen for the low frequencies. To appreciate the interest in the open-loop method, consider a rigid airplane having only vertical and pitch degrees of freedom and equipped with two controls: a conventional elevator with deflection β and a direct lift control system with deflection σ. The movement is determined, with evident notation, by the reduced equation:

$$(22)\quad \begin{cases} m\dfrac{\ell^2}{V^2}\xi'' + \dfrac{\ell}{V}\xi' - \theta = \dfrac{w}{V} + \lambda\beta + \lambda'\sigma \\[2ex] -\dfrac{\ell\dot{\xi}}{V} + c\,\dfrac{\ell^2\theta''}{V^2} + \eta\,\dfrac{\ell\dot{\theta}}{V} + \theta = -\dfrac{w}{V} - \nu\beta + \nu'\sigma \end{cases}$$

(where ℓ is a reference length and ξ is a reduced displacement $\frac{z}{\ell}$).

It may be seen then that it is sufficient to measure onboard the airplane the turbulence w which it encounters and to choose the control laws:

$$\beta = k_1 \frac{w}{V} \quad , \quad \sigma = k_2 \frac{w}{V}$$

such that the right hand side of the equations is cancelled, to completely suppress the responses of the airplane to turbulence. The natural frequencies and dampings associated with mechanics of flight will remain unchanged. As a result, if the airplane has good flying qualities it will not be changed at all by the presence of the system. It is evident that this is an ideal example since one assumes a rigid airplane, perfectly effective controls, and cylindrical turbulence.

This last point is probably the most important; in effect, in an open-loop system one is required to measure "the turbulence encountered by the airplane." It is necessary that this expression has a meaning, that is to say, that one is able to consider that an instantaneous measurement of w will be representative of the turbulence encountered. The discussion of paragraph 2.2 gives the limits of such a hypothesis, that it is admissible only at low frequencies. In consequence, the open-loop optimization will never be able to be employed to control the flexible modes, for control of which one should utilize the American method of counter-reaction. Its very important role consists of alleviating, better than by any other approach, the behavior of the airplane in the domain of flight mechanics.

Example of application of the method

In the ideal example which has been presented, the rigid airplane has at its disposal two independent controls, pitch and lift, which allow complete determination of two degrees of freedom. It would not have been thus if the airplane had been considered as flexible, or if only one control surface had been utilized for control. It is to this latter case, which has been the object of several publications, References (25), (26), and (27), that we now give attention. We will show what improvement can be obtained by an open-loop system on a delta wing airplane controlled only by its conventional elevons ($\sigma = 0$).

In this example, which corresponds to a Mirage III flying at low altitude and at great speed, the airplane can be considered rigid, since its first natural frequency is about 9 cycles per second. Equations (22) with $\sigma = 0$, then represent its behavior. Also, the measure of $w(t)$ at a point can be considered as representative of the turbulence encountered in the range 0-4 c/s.

If one calls $\beta(t)$ the movement of the surface which serves for control, $a(t)$ the impulsive response of a parameter $q(t)$ of the airplane to the reduced turbulence $\frac{w}{V}$, and $b(t)$ the impulsive recponse to deflection of the control, the problem of optimization appears as follows: "Find a function $k(t)$ physically realizable (that is to say, the impulsive response of a stable system) such that the control law:

$$(23) \qquad \beta(t) = k(t) * \frac{w(t)}{V}$$

mininizes the variance of the response:

$$(24) \quad q(t) = a(t) * \frac{w(t)}{V} + b(t) * \beta(t)$$

the variance should be calculated in the range (0-4 c/s)"

(* represents a convolution).

Designating by capitals the Fourier transforms of the functions a, b, and k, we write:

$$S_w(\omega_R) = \frac{r}{\pi}\,\sigma_w^2\,\Psi_w(\omega_R) \quad ; \quad r = \frac{\ell}{L}$$

et $$\omega_R = \frac{\omega \ell}{V}$$

the problem admits of an equivalent formulation: "Find a transfer function $K(i\omega)$ which is physically realizable such that":

$$8\int_0^{+\infty} \left| A(i\omega_R) + B(i\omega_R) K(i\omega_R) \right|^2 \left| \sigma(i\omega_R) \right|^2 \Psi_w(\omega_R) \, d\omega_R = 0 \qquad (25)$$

$G(i\omega_R)$ represents the transfer function of a filter which in practice limits the calculation of the variance to the band of frequency (0-4 c/s).

Equation (25) is solved by the method of the integral equation of Wiener, from which one deduces the form of the control law:

$$K(i\omega) = \frac{1}{\lambda}\left[C_0 + \frac{C_1 \frac{V}{\ell}}{i\omega \cdot s_1 \frac{V}{\ell}} + \frac{C_2 \frac{V}{\ell}}{i\omega \cdot s_2 \frac{V}{\ell}} + \frac{C_3 \frac{V}{\ell}}{i\omega \cdot s_3 \frac{V}{\ell}} \right] \qquad (26)$$

The coefficient C_i and the nondimensional poles s_j do not depend explicitly on the flight speed. They are uniquely expressed as a function of the density of the air and the dimensionless aerodynamic parameters. It is noted that formula (26) defines the condition of adaptation of an automatic pilot to different regimes of flight. In Reference (27), where all the calculations which lead to formula (26) are developed, is established, for a Mirage III, the variation of the improvement as a function of the different parameters (one recalls that the improvement is defined as the ratio of the variance of the unpiloted airplane to the variance of the piloted airplane in the presence of the same turbulence). Figure 15 represents the variation of the improvement in acceleration of the center of gravity as a function of the cutoff frequency chosen for the calculation of the variance.

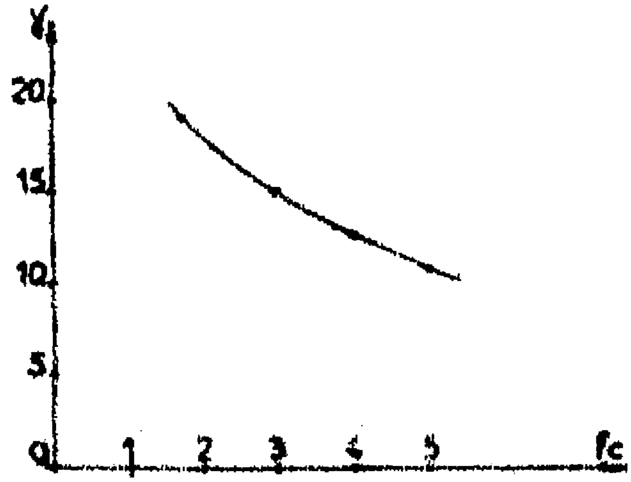


Fig. 15

EVOLUTION DU GAIN γ EN FONCTION DE LA FRÉQUENCE DE COUPURE f_c.

(Variation of Improvement γ as a Function of Cut-off Frequency f_c)

Figure 16 shows the influence of the scale of turbulence on the effectivenss of the automatic pilot.

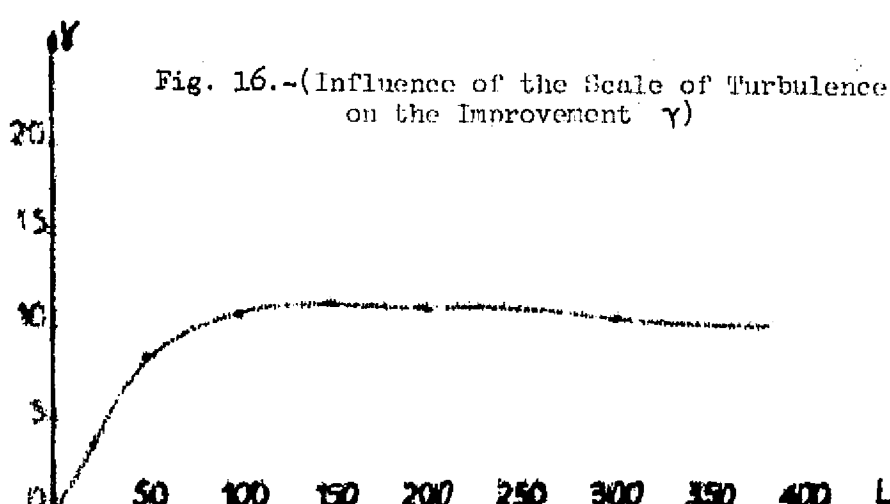


Fig. 16.-(Influence of the Scale of Turbulence on the Improvement γ)

Fig.16 INFLUENCE DE L'ÉCHELLE DE LA TURBULENCE SUR LE GAIN γ

It appears that the improvements which may be hoped for from the system are, for reasonable scales and for the band pass chosen, of the order of 10, which cannot be obtained by any closed-loop system.

The implementation of such an automatic pilot in the Mirage III is actually in progress. Control laws have been verified on analog computers (taking into account limits of authority and nonlinearities of the servomechanisms) and in a flight simulator; the apparatus for calculating the turbulence has been installed onboard the airplane and tested in flight. The "black boxes" are completed and the first flight tests of the system will probably have been carried out by the time when this conference is presented.

Conclusions

As has been emphasized since the introduction, the presentation does not pretend to be exhaustive. Various subjects have been introduced: modeling of the atmospheric turbulence, knowledge of the transfer functions, and optimization of flight in turbulence, subjects on which we are now going to attempt to reach conclusions.

In the domain of practical modeling of atmospheric turbulence the model of Press, where the atmosphere is supposed to be formed of regions which are stationary and Gaussian, has been utilized for about 15 years, in conjunction with discrete gust models; although very useful, the model of Press is arbitrary in its conception and is poorly suited to the prediction of extreme loads.

To alleviate these difficulties two new approaches are proposed: One, due to Jones, attempts to tie down the form and statistics of discrete gusts in such a way that the spectral density of the turbulence is correctly represented in the inertial domain. The other suggests considering the packets of turbulence of amplitude greater than a given value as independent events distributed according to a Poisson process. Much work is still required to lead to a logical presentation of these two proposals.

The calculation of transfer functions of a flexible airplane in turbulence is now a routine matter for manufacturers for the case of a representation of the turbulence by waves which are constant across the span. Thanks to the notion of transverse correlation length associated with the Karman model, we have been able to find the limits of validity of this hypothesis. Calculations carried out on many modern airplanes show in effect that the representation by waves constant across the span is only valid in the domain of frequencies associated with mechanics of flight. When flexible modes intervene, one should take into account the isotropy of a turbulent field, which hardly introduces any new complications at all in the chosen method of calculations. The comparisons between calculated transfer functions and measured transfer functions still show sufficient disagreement that measurement in flight are often judged necessary. The presentation, which recalls the methods of identification adapted to measurement of transfer functions in turbulence, emphasizes the simplicity and the low cost of such an undertaking.

In the domain of optimization of flight in turbulence, the majority of the work has been carried out previously in the United States, based on the conception of systems of counter-reaction in which suitable responses picked up in the airplane are reinjected in the controls. Numerous projects have been carried out with this approach and the system has flown with excellent results in a B-52. Parallel to these American undertakings, ONERA has proposed for several years an open-loop method where the turbulence measured onboard the airplane serves to add to the control movement. This new approach, which gives best results in the regime of the mechanics of flight, is unfortunately not very suitable for control of flexible modes because of the fact that at frequencies corresponding to turbulence measured instantaneously, it is only poorly representative of the excitation actually undergone by the airplane. As a result one is led to a mixed concept: the ONERA open-loop method in the mechanics of flight domain where the maximum energy should be absorbed and the American method of counter-reaction for the flexible modes.

Finally, the author hopes that his presentation will permit the appreciation of the importance of problems which remain to be solved and on which work has been started in only a few cases. There is still a place for young talent in the field of flight in turbulence.

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